THE INSTITUTION OF ENGINEERS, SRI LANKA
IESL ENGINEERING COURSE

PART III(A) EXAMINATION—NOVEMBER 2008

323 CONTROL SYSTEMS

Instructions: Answer **five** questions **only**, selecting not more than two from each section. Tentative mark allocation for each part is shown in brackets for guidance only. Semilog papers are provided.
Time: Three hours.

SECTION A

Q1. (a) Briefly describe the following methods of determining stability of control systems. **[6 marks]**

i. Routh-Hurwitz criterion.

ii. Nyquist criterion.

iii. Bode diagram.

(b) Determine if the systems with the following closed-loop transfer functions are stable, marginally stable, or unstable, by inspection. **[4 marks]**

i. $M(s) = \dfrac{1}{(s+1)(s+2)}$.

ii. $M(s) = \dfrac{10(s-1)}{(s+1)(s^2+9)}$.

(c) Use the Routh-Hurwitz criterion to determine the range of values of K for which the system with the following transfer function is stable. **[10 marks]**

$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2+s+1)(s+2)+K}.$$

Q2. (a) What is *compensation* in relation to a control system? **[3 marks]**

(b) Show that the characteristic equation for the system shown in Fig. Q2(b) can be written as

$$1 + \frac{Ks}{(s+j2)(s-j2)(s+5)} = 0,$$

where $K = 20k$. **[5 marks]**

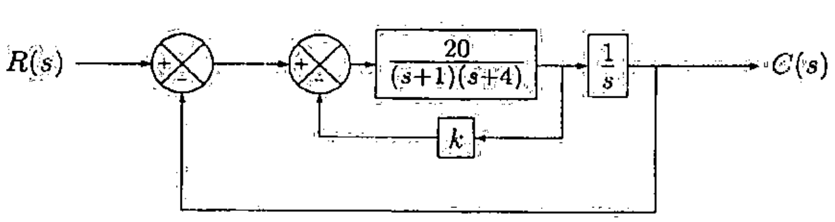

Figure Q2(b): System for Q2(b).

98

(c) Draw a root locus diagram for the system shown in Fig. Q2(b), and determine the value of k, such that the damping ratio of the dominant closed-loop pole is 0.4. **[12 marks]**

Q3. (a) Using the mathematical expression, briefly explain the proportional-plus-integral-plus-derivative controller. **[4 marks]**

(b) Simplify the block diagram shown in Fig. Q3(b), and obtain the corresponding closed-loop transfer function. **[6 marks]**

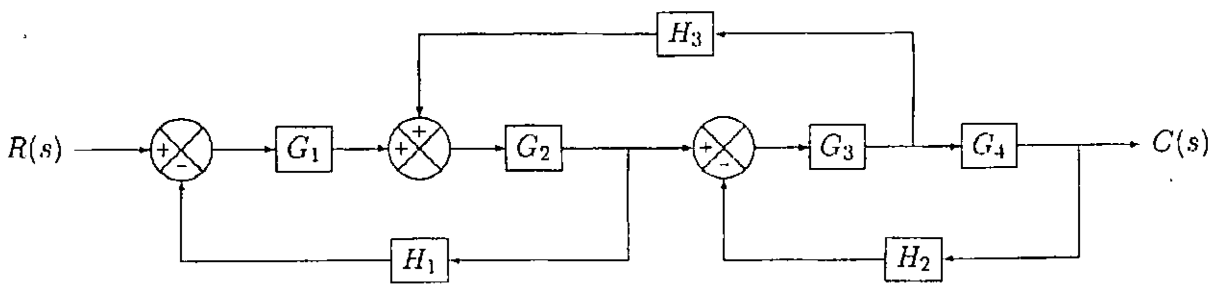

Figure Q3(b): System for Q3(b).

(c) Consider the system represented by the block diagram shown in Fig. Q3(c). **[10 marks]**

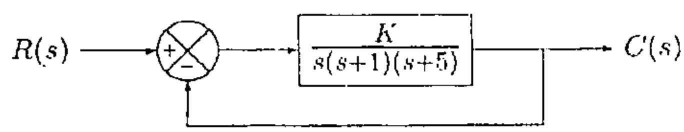

Figure Q3(c): System for Q3(c).

i. Draw the Bode diagram for $K = 10$.

ii. Comment on the stability of the system when $K = 10$ and $K = 100$.

SECTION B

Q4. (a) Compare conventional control theory and state-space based modern control theory. **[4 marks]**

(b) A spring-mass-damper system is shown in Fig. Q4(b). k, m, and c are the usual parameters of the system. The external force $u(t)$ is the input to the system and $y(t)$, the displacement of the mass, is the output. The displacement $y(t)$ is measured from the equilibrium position in the absence of the external force. **[10 marks]**

i. Find the state-space equations of the system in the standard matrix-vector form.

ii. Starting from the state-space equations above, show that the system transfer function is

$$G(s) = \frac{1}{ms^2 + cs + k}.$$

(c) Consider the system given by **[6 marks]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

108

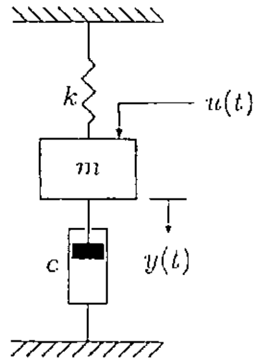

Figure Q4(b): System for Q4(b).

and

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

State whether this system is

i. completely state controllable,
ii. output controllable. and
iii. completely observable.

Q5. (a) Consider the following transfer function: [8 marks]

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}.$$

Derive the following controllable canonical form of the state-space representation of this transfer function:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-1} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} b_n - a_n b_0 & b_{n-1} - a_{n-1} b_0 & \cdots & b_1 - a_1 b_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u.$$

(b) Obtain the controllable canonical form of the following transfer function: [4 marks]

$$\frac{Y(s)}{U(s)} = \frac{2s^3 + s^2 + s + 2}{s^3 + 4s^2 + 5s + 2} = \frac{2s^3 + s^2 + s + 2}{(s+1)^2(s+2)}.$$

(c) Obtain the Jordan canonical from (JCF) of the system in Q5(b) above, and comment on the controllability of the system based on the JCF. [8 marks]

Q6. (a) Find the solution $x(t)$ of the differential equation [6 marks]

$$\ddot{x} + 3\dot{x} - 2x = 0, \quad x(0) = a, \quad \dot{x} = b.$$

105

(b) Considering the homogeneous state equation $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$, show that the state transition matrix is [7 marks]

$$\Phi(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}].$$

(c) Obtain the response $y(t)$ of the following system: [7 marks]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where $u = u(t)$ is the unit step.

SECTION C

Q7. (a) What are the causes of nonlinearities in control systems? [4 marks]

(b) Obtain the phase plane trajectory of the the differential equation. [6 marks]

$$\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right) = 0,$$

with the initial conditions $x(0) = 0$ and $(dx/dt)|_{t=0} = 1$.

(c) Determine the sufficient conditions in terms of the ratio $f(x_1)/x_1$ for the stability of the system, [10 marks]

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} f(x_1),$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Q8. (a) Define the terms BIBO stability and zero-input stability in relation to a linear time-invariant discrete-data system. [2 marks]

(b) State the stability condition for the following closed-loop transfer functions: [6 marks]

i. $M(z) = \dfrac{z}{(z-0.2)(z-0.8)}$

ii. $M(z) = \dfrac{5z}{(z+1.2)(z-0.8)}$

iii. $M(z) = \dfrac{(z+1)}{z(z-1)(z-0.8)}$

(c) Consider the sampled-data system with a zero-order hold (ZOH) shown in Fig. Q8(c). Show that the open-loop transfer function is [12 marks]

$$G(z) = \frac{0.368K(z+0.717)}{(z-1)(z-0.368)}.$$

Find the range of values of K such that the system is stable, using Routh's criterion.

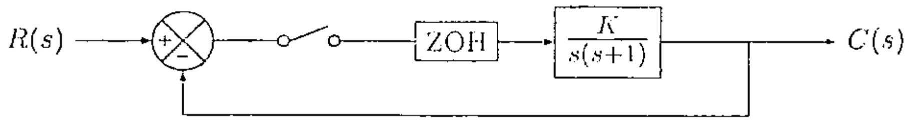

Figure Q8(c): System for Q8(c).

Q9. (a) What are the advantages of using programmable logic controllers (PLCs)? **[3 marks]**

(b) List the functional components in a PLC, and describe three of them. **[5 marks]**

(c) Fig. Q9(c) shows a schematic diagram of a hydraulic hoist. Design a program using ladder diagram logic to operate the hoist as per the following specifications: **[12 marks]**

i. The *start* push-button starts the *pump motor*, and the *pump motor* stays ON until the *stop* push-button is pressed. A *light* indicates that the *pump motor* is ON.

ii. The hoist can be turned OFF at any time using the *stop* push-button.

iii. Using a 3-position, 4-way valve operated by two solenoids *sola* and *solb*, the piston can be moved up, down, or locked in position.

iv. When the *pump motor* is ON, pressing the push-button *up*, connects the hydraulic supply line to the bottom of the cylinder and the return line to the top of the cylinder. The pump forces hydraulic fluid in to the bottom of the cylinder and the piston moves up. Solenoid *sola* is ON in this stage.

v. When the *up* button is released, i.e., when neither *sola* nor *solb* is ON, the piston is locked in place.

vi. When the *pump motor* is ON, pressing the push-button *down* connects the hydraulic supply line to the top of the cylinder and the return line to the bottom of the cylinder, and the piston moves down.

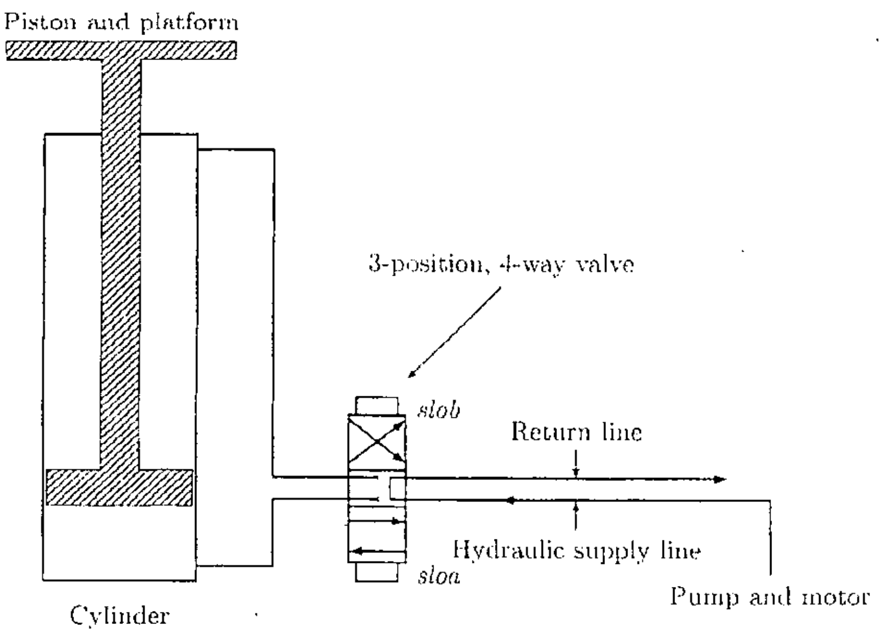

Figure Q9(c): System for Q9(c).

103