THE INSTITUTE OF ENGINEERS-SRI LANKA

PART II EXAMINATION- JUNE 2012

201 MATHEMATICS

Answer **FIVE** Questions only **Time Allowed: Three Hours**

Question 1

(a) Suppose that f is the function given by $f(x,y) = xe^{x^2/y^2}$

Determine the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. Verify that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f(x,y)$

(b) The function f is given by $f(x,y) = y^3 - x^3 - 2xy + 5$.

Determine the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

Show that the points (0, 0) and (2/3,−2/3) are the only critical (or stationary) points of f.

(c) Determine whether each of these points is a local maximum, local minimum, or saddle point.

Question 2

(a) Use the method of Lagrange multipliers to optimise the function $f(x,y) = x^{3/8}y^{2/3}$ subject to the constraint $x^2 + y^2 = 25$ where $x,\ y > 0$.

By sketching the constraint and some contours of f, justify your use of the method of Lagrange multipliers and determine whether the point you have found maximises or minimises f subject to the constraint.

(b) Find the fourth -order Maclaurin series for sin x and e^x.Hence or other wise find Maclaurin series of $e^{\sin x}$ up to fourth order

Question 3

Let $A = \begin{pmatrix} -1 & 1 & 2 \\ -6 & 2 & 6 \\ 0 & 1 & 1 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Show that the vector v is an eigenvector of A. What is the corresponding eigenvalue? Find the other eigenvalues of A, and an eigenvector for each of them. Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Question 4

(a) A function $f: R^3 \rightarrow R$ is defined by

$f(x, y, z) = \ln(xy + z)$:

(a) Find the gradient of f at the point (a, b, c).

(b) Verify that the point (1, 1, 0) is on the surface $\ln(xy + z) = 0$; and find the normal vector and the tangent plane to the surface at this point.

(c) Consider the points, (x, y, z), at which the rate of increase of f in the direction $(x/2, y/2, z)^T$ is equal to two. Show that all of these points lie on the surface with equation

$x^2 + y^2 + 4z^2 = 1$

Question 5

Show that a vector function $\underline{A} = (2xy + 2y)\underline{i} + (x^2 + 2x)\underline{j} + 3z^2\underline{k}$ is conservative.

(b) Find scalar potential of ϕ such that $\underline{A} = \nabla\phi$.

(c) Find work done by the force $\underline{A}$ from the point (1,1,2) to (1,3,4).

Verify Divergence Theorem, given that $\underline{F} = 4x\underline{i} - y\underline{j} + z\underline{k}$ and S is the surface of the spherical surface bounded by the plane $z = 0$, and $x^2 + y^2 + z^2 = 4$.

Question 6

(a) If $\underline{r}$ is a position vector of point P in 3 dimensional Space, then;

Show that (i) Div($\underline{r}$)=3,

(ii)Curl($\underline{r}$)=0,

(iii) $\text{div}(r^n r) = (n+3)r^n$,

(b) Find the Laplace Transform of the followings:

(i) $t^2 + 5t$ (ii) t^3e^{-3t} (iii) $\sin t$

(c) Find the inverse transform of the followings:

(i) $\frac{1}{(s+2)^4}$ (ii) $\frac{1}{(s^2+4)}$

Question 7

(a) A company has two machines which produce cupboards. 75% are produced by the new machine and the remaining 25% by the older machine. In addition, the new machine produces 8% defective cupboards. The old machine produces 23% defective ones.

(i) What is the probability that a randomly chosen cupboard produced by the company is defective?

(ii) Given that a randomly chosen cupboard has been chosen and found to be defective, what is the probability it was produced by the new machine?

(b Cooking sauces are sold in jars containing a stated weight of 500 g of sauce The jars are filled by a machine. The actual weight of sauce in each jar is normally distributed with mean 505 g and standard deviation 10 g.

(i) Find the probability of a jar containing less than the stated weight.

(ii) In a box of 30 jars, find the expected number of jars containing less than the stated weight.

The mean weight of sauce is changed so that 1% of the jars contain less than the stated weight. The standard deviation stays the same.

(iii) Find the new mean weight of sauce.

Question 8

A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round.

The random variable X is the number of points Linda scores in a round.

(b) Find the probability distribution of X.

(c) Find the mean and the standard deviation of X.