THE INSTITUTION OF ENGINEERS, SRI LANKA
IESL ENGINEERING COURSE

PART III EXAMINATION—JUNE 2010

323 CONTROL SYSTEMS ENGINEERING

Instructions: This question paper contains nine questions in 7 pages. Answer **five** questions **only**. Tentative mark allocation for each part is shown in brackets for guidance only. **Time:** Three hours.
Additional material: Semi-log graph papers are provided.

Q1. (a) What is a closed-loop control system? Explain the same system by analyzing how a driver navigates a car along the road. [**6 marks**]

(b) Determine the stability of a unity feedback system with an open-loop transfer function given by [**6 marks**]

$$G(s) = \frac{10}{s(s-1)(2s+3)}.$$

(c) Use Routh's stability criterion to determine the stability of a system with the characteristic equation [**8 marks**]

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0.$$

You must show Routh's array as a part of your answer.

Q2. (a) Why is root locus considered to be an important technique in rehalation to control systems?. [**4 marks**]

(b) The open-loop transfer function of a unity feedback system is [**10 marks**]

$$G(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+16)}.$$

Use Routh's criterion to find the values of K at which the root loci cross the imaginary axis, sketch the root locus, and find the values of K for which the system is stable.

(c) Figure 1 represents a system whose root locus plot is shown in Figure 4. Show that the characteristic equation can be written as [**6 marks**]

$$1 + \frac{K}{s(s^2+9s+18)} = 0,$$

where $K = 10a$. Determine the value of a, such that the damping ratio of the dominant closed pole is $\xi = 0.5$. Attach Figure 4 to your answer script.

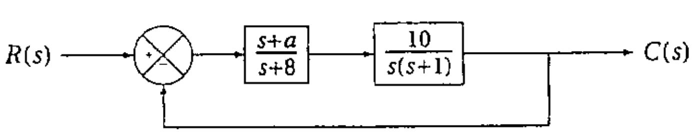

Figure 1: Block diagram for Q2c.

Q3. (a) Frequency repossess analysis techniques, such as Bode diagram, can only give an indirect indication of the the system's transient response. Briefly explain the aforementioned sentence, and list methods of indirectly specifying the transient response in frequency domain analysis techniques. **[6 marks]**

(b) Use Nyquist criterion to investigate the stability of a closed-loop system with the open loop transfer function **[6 marks]**

$$G(s)H(s) = \frac{K(s+3)}{s(s-1)}, \quad K > 1.$$

(c) Consider a system with the open-loop transfer function **[8 marks]**

$$G(s) = \frac{4}{s(s+2)}.$$

A lead compensator is of the form

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1}.$$

Design such a compensator for the system so that the static velocity error constant K_v is $20\,\mathrm{s}^{-1}$, the phase margin is at least 50°, and the gain margin is at least 10 dB.

Q4. (a) Figure 2 shows a two-rolling-cart-cart system. M_1 and M_2 are the masses of the first and the second cart, respectively. k_1 and k_2 are the spring constants, and b_1 and b_2 are the damping coefficients. Use the assignment $x_1 = p$, $x_2 = q$, $x_3 = \dot{p}$, and $x_4 = \dot{q}$, and obtain the state-space model of this system. **[6 marks]**

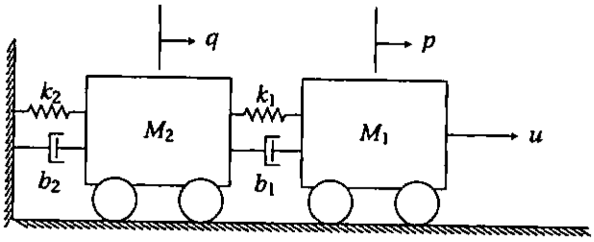

Figure 2: System of two rolling carts for **Q4a**.

(b) Give the definitions of controllability and observability, and interpret them in your own words. **[6 marks]**

(c) Two state-space representations of a system is given by the following equations: **[8 marks]**
Representation 1:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -0.4 & -1.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u,$$

$$y = \begin{bmatrix}0.8 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

Representation 2:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & -0.4\\ 1 & -1.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0.8\\ 1\end{bmatrix}u,$$

$$y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

i. Show that both the representations result in the same transfer function.
ii. determine the observability and controllability of the two representations.
iii. Do observability and controllability depend on the representation?

Q5. (a) Consider control system, in regular notation, **[6 marks]**

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

Draw the block diagram of the closed-loop control system with $u = -Kx$, with thick lines representing vector paths.

(b) What is the main condition for full-state feedback control design? **[4 marks]**

(c) Consider a regulator system with the plant given by **[10 marks]**

$$\dot{x} = Ax + Bu,$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}, \quad \text{and } B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The system uses state feedback control $u = -Kx$. If the closed-loop poles are placed at $s = -2 \pm j2$, determine the required state variable feedback gain matrix K. Assume that the complete state vector is available for feedback.

Q6. (a) Consider the homogeneous state equation **[6 marks]**

$$\dot{x}(t) = Ax(t).$$

Show that

$$x(t) = e^{At}x(0).$$

(b) Consider the following system: **[6 marks]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Show that the state transition matrix of the system is

$$\Phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}.$$

(c) For the system **[8 marks]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

where $u(t)$ is the unit step function occurring at $t = 0$, find

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

if $x_1(0) = 0$ and $x_2(0) = 0$.

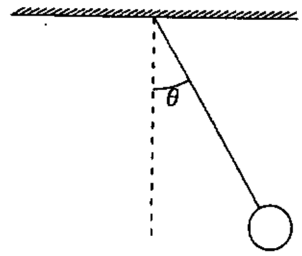

Figure 3: Figure for Q7a.

Q7. (a) The motion of a pendulum of length l, as shown in Figure 3, is given by **[6 marks]**

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0.$$

Briefly describe why the equation is said to be nonlinear and linearize it about $\theta = 0$.

(b) Find the phase plane trajectory of the solution of the differential equation **[6 marks]**

$$\frac{d^2x}{dt^2} + x = 0,$$

with initial conditions $x(0) = 0$ and $\left.\frac{dx}{dt}\right|_{t=0} = 1$.

(c) Show that, for the system represented by **[8 marks]**

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + \left(\frac{dx}{dt}\right)^3 + x = 0,$$

the origin is stable.

Q8. (a) What are the advantages of digital control? **[6 marks]**

(b) Consider the system with the process transfer function **[8 marks]**

$$G_p(s) = \frac{100}{s^2 + 100}.$$

This is preceded by a zero order hold module, sampling with a period of $T = 0.05$s. Derive the transfer function and determine whether the digital system is stable.

(c) The characteristic equation of a sampled system is given by **[6 marks]**

$$z^2 + (K-2)z + 0.75 = 0.$$

Use the root locus method to find the range of K so that the system is stable.

Q9. (a) What are the advantages of PLC-based control in comparison to hardwired control? **[4 marks]**

(b) Briefly describe the use of the following devices, giving examples where possible: **[4 marks]**

i. limit switches.

ii. proximity switches.

iii. pressure or vacuum switches.

iv. push buttons.

v. relay coils.

(c) Use ladder diagram logic to design a controller for for a bi-directional motor. Assume that when the starter contact $M1$ is energized, the motor spins in the forward direction and when the starter contact $M2$ is energized it spins in the reverse direction. Notice that both $M1$ and $M2$ cannot be energized simultaneously. Starting the motor to spin in the forward direction must be doable by momentarily pushing the the push-button *forward*. Starting the motor to spin in the reverse direction must be doable by momentarily pushing the the push-button *reverse*. When the motor is spinning in one direction, it must not be forced to spin in the opposite direction. Therefore, a time delay of t must be introduced between direction reversals. Provide two pilot lamps, green when the motor is spinning in the forward direction, and blue when the motor is spinning in the reverse direction. The motor must be stoppable by pushing a push-button *stop*, and there must be overload protection. In this design, you must exercise good design principles. Briefly explain each step or rung of your design. [**12 marks**]

Table of Laplace and z-Transforms as Applicable to Conrod Systems

	$x(t)$	$X(s)$	$X(z)$
1.	$\delta(t) = \begin{cases} \frac{1}{\epsilon}, & t < \epsilon, \epsilon \to 0 \\ 0 & \text{otherwise.} \end{cases}$	1	
2.	$\delta(t-a) = \begin{cases} \frac{1}{\epsilon}, & a < t < a+\epsilon, \epsilon \to 0 \\ 0 & \text{otherwise.} \end{cases}$	e^{-as}	
3.	$\delta_o(t) = \begin{cases} 1, & t = 0 \\ 0 & t = kT,\ k \neq 0. \end{cases}$		1
4.	$\delta_o(t-kT) = \begin{cases} 1, & t = kT \\ 0 & t \neq kT. \end{cases}$		z^{-k}
5.	$u(t)$	$\frac{1}{s}$	$\frac{1}{1-z^{-1}}$
6.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
7.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
8.	$1-e^{-at}$	$\frac{1}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
9.	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
10.	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$
9.	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
10.	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z(z-e^{-aT}\cos(\omega T))}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$

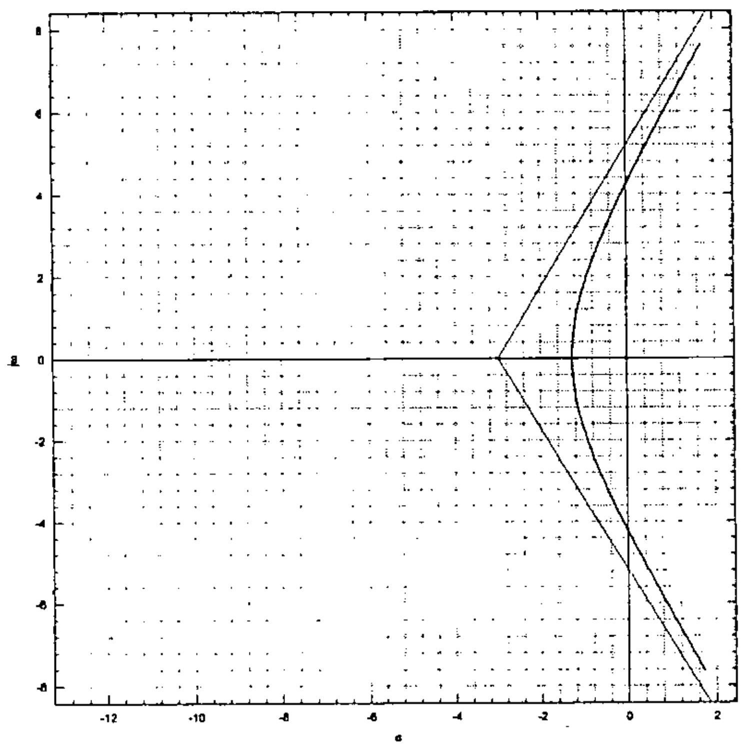

Figure 4: Root locus plot for **Q2c** to be attached to the answer script.