THE INSTITUTE OF ENGINEERS-SRI LANKA

PART III(A) EXAMINATION- SEPTEMBER 2011

301 MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) State Cauchy's Integral formula.
(b) Find the value of each of the following integrals by using Cauchy's Integral formula:

(i) $\oint\limits_{|z|=2} \frac{z^2+z+1}{z-3}dz$ (ii) $\oint\limits_{|z|=2} \frac{e^z}{z-1}dz$ (iii) $\oint\limits_{|z|=1} \left(z+\frac{1}{z}\right)^2 dz$

(b) Find the image of the region bounded by a circle of radius 4, a circle of radius 6, line $\frac{\pi}{4}$, line $\frac{\pi}{2}$, under mapping $W = Z^2$. Sketch the region on z plane and its image on W plane.

Question 2

(a) List and classify if they exist, the singularities of the following functions giving the orders of the poles,

(i) $f(z) = \frac{1}{z^2} + \frac{1}{z^2+1}$, (ii) $f(z) = \frac{1-\cos z}{z}$ (iii) $f(z) = \frac{1}{\exp(z^2)-1}$

(b) Find the residues of the following functions at the points stated; [res f(z) at z = a with pole of order m is given as $\lim\limits_{z\to a} \frac{d^{m-1}}{dz^{m-1}}\left[f(z)(z-a)^m\right]$

(i) $\frac{\sin z}{z^2}$, at $z = 0$, (ii) $\frac{1}{(z-1)^2(z+1)}$, $z = 1$; (iii) $\frac{1}{(1-\cos z)}$, at $z = 0$,

(c) Use Cauchy Residue theorem to find the value of the integral: $\int\limits_C \frac{dz}{(z+1)(z-2)^2}$, where c is a rectangle with vertices $-3+i,\ -3-i,\ 3+i,\ 3-i$.

Question 3

Find by the method of separation of variables the solution of u(x,t) of the boundary value problem $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, $t > 0, \quad 1 \le x \le 2$

$$u(0,t) = u(2,t) = 0, \ t > 0$$

$$u(x,0) = x, \quad 1 \le x \le 2$$

[Fourier half range sine series for f(x), $0 \le x \le L$, is given as

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}, \quad \text{where } b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx \quad]$$

Question 4

(a) Suppose the scores x, on a college entrance examinations are normally distributed with mean of 550 and standard deviation of 10. A certain prestigious university will consider for admission only those applicants whose scores exceed the 90^{th} percentile of the distribution. Find the minimum score an applicant must achieve in order to receive consideration for admission to the University.

(b) A manufacturer of car exhaust pipes wishes to check the achievement of a standard in which their average length of life is longer than one year (otherwise they have to be replaced under guarantee) but no longer than 18 months under normal use. From a pilot sample he computes the standard deviation as 7 months .What minimum size of sample will he require to check the achievement of the desired standard with 95% confidence.?

Question 5

(a) Assume that the average annual income for government employees in any nation is reported by the Census Bureau to be $18,750.00. There was some doubt whether the average yearly income of government employees in Washington was representative of the national average. A random sample of 100 government employees in Washington was taken and it was found that their average salary was $19,246.00 with a standard deviation of $2,610.00. At a level of 5% significance, can we conclude that the average salary of government employees in Washington is representative of the national average?

Question 6

(a)Two opinion polls, in which random sample of 1,600 and 2,000 electors respectively were asked about the voting intentions in the general election, showed 48% and 51% respectively in favor of the ruling political party. Is this a significance difference at 5% level of significance.?

(b) Compute the combined percentage from the two opinion polls above and say whether or not it represents a significant increase over vote of 44% in the previous general election for the ruling party at 5% level of significance.?

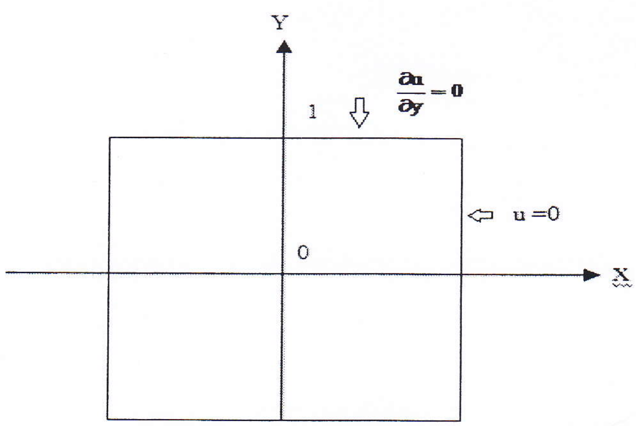
Question 7

(a) The current in electrical winding gives rice to an internal generation of heat that is dissipated by radiation from the boundaries. The steady temperature of a cross section is given by the equation: $\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial y^2}=-16y$ inside the square, $x=\pm 1,\ y=\pm 1$.

(as shown in the following figure)

The temperature distribution u, also satisfy the boundary condition u =0 on x = 1, $\frac{\partial u}{\partial y}=0$ *on* $y=1$, also temperature distribution is symmetric with respect to Ox and Oy. Find temperature distribution of u. By the use of the final difference method with uniform grid length 1/3, obtain a system of linear equations for a numerical solution of this problem.

Describe briefly how the linear system derived can be easily solved.



Question 8

Given that the temperature distribution u(x,t) along a rod at distance x at time t satisfies the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that end of the rod are kept in contact with blocks of melting ice and that the initial temperature distribution is

$u = 2x, \quad 0 \le x \le 1/2$

$u = 2(1 - x), \quad 1/2 \le x \le 1$

find the numerical solution of temperature u for x = (1/4)*i ; i = 1, 2, 3; t = (1/4)*j, j= 1, 2, 3.

Also show how a numerical solution can be found for x= (1/4)*i; i= 1, 2, 3; t = (1/2)*j, j= 1, 2.

Question 9

Given an initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$, the second order Runge-Kutta method gives an approximate value for y at $x_1 = x_0 + h$ as $y(x_0) = y_0 + \frac{1}{2}(k_1 + k_2)$, where

$k_1 = hf(x_0, y_0),\ k_2 = hf(x_0 + h, y_0 + k_1)$.

Use this method to solve the equation: $\frac{dy}{dx} = xy + 2$,

$y(0) = 2$ to find y at x = 0.1 and x = 0.2

Write a simple programme to solve the above differential for y at x= 0.1, 0.2, …, 1.0 and plot results as a graph.
