THE INSTITUTE OF ENGINEERS-SRI LANKA

PART III

EXAMINATION- SEPTEMBER 2012

301 MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) (i) Show that $u = u(x, y)$ is harmonic in some domain and find harmonic conjugate $v = v(x, y)$ so that $f(z) = u + iv$ be an analytic function, if $u = \sinh x \sin y$.

(ii) Find the derivative of u and v with respect to x and y partially, if

$u - v = (x - y)(x^2 + 4xy + y^2)$ hence find $f(z) = u + iv$ in terms of z.

(b) Evaluate the following contour integration by using Cauchy Residue theorem:

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$$

Question 2

(a) State Cauchy integral formula.

(b) Use Cauchy integral formula to evaluate:

(i) $\oint_C \frac{dz}{(z-1)(z-2)}$, Where C is the circle $x^2 + y^2 = 9$.

(ii) $\oint_C (z^2 - 2z + 1)dz$, Where C is the circle $x^2 + y^2 = 9$.

Question 3

Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, $t > 0$, $0 \le x \le \pi$ satisfying the following boundary equations;

(i) $\frac{\partial u}{\partial x}(0,t) = 0$ for all $t \ge 0$

(ii) $\frac{\partial u}{\partial x}(\pi,t) = 0$ for all $t \ge 0$

(iii) $u(x,0) = \cos^2 x \quad 0 \le x \le \pi$

Question 4

(a) On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $1/2$, $1/6$, and $1/3$ respectively. The probability of being late when using these methods of travel is $1/5$, $2/5$, and $1/10$ respectively.

(a) Draw a tree diagram to represent this information.

(b) Find the probability that on a randomly chosen day

(i) Bill travels by foot and is late,

(ii) Bill is not late.

(c) Given that Bill is late, find the probability that he did not travel on foot.

Question 5

(a)The mean lifetime of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If it is the mean lifetime of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$ hours, using 5% level of significance.

(b) At an agricultural station it was desired to test the effect of a given fertiliser on wheat production. To accomplish this, 24 plots of land having equal areas were chosen; half of these were treated with the fertiliser and the other half were untreated (control group). Otherwise the conditions were the same. The mean yield of wheat on the untreated plots was 131 kilograms with a standard deviation of 11 kilograms, while the mean yield on the treated plots was 139 kilograms with a standard deviation of 10 kilograms. Can we conclude that there is a significant improvement in wheat production because of the fertiliser if a significance level of 5% is used?

Question 6

The weight, y grams, and the length, x mm, of 10 randomly selected newborn turtles are given in the table below.

x	49.0	52.0	53.0	54.5	54.1	53.4	50.0	51.6	49.5	51.2
y	29	32	34	39	38	35	30	31	29	30

(You may use $S_{xx} = 33.381$, $S_{xy} = 59.99$, $S_{yy} = 120.1$)

(a) Find the equation of the best fitted line of y on x in the form $y = \alpha + \beta x$.

(b) Use your regression line to estimate the weight of a newborn turtle of length 60 mm.

(c) Comment on the reliability of your estimate giving a reason for your answer.

Question 7

Calculate the finite difference solution of the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t \ge 0$

Satisfying the initial and the boundary conditions

$u = \sin \alpha x$ at $t = 0$, $0 \le x \le 1$,

$u = 0$, at $t = 0$ and $x = 1$ for $t > 0$

Use an explicit method with $h = 0.1$ and $\alpha = 0.1$, to find u, up to $t = 0.003$.

Question 8

(a)Apply fourth order Runge- Kutta method to find an approximate value of y when $x = 1.1$,

given that $\frac{dy}{dx} = x - y, \quad and \quad y = 1 \quad at \quad x = 1.$

(b) Compare with solution of (a) and exact value of y

[With usual notations for $\frac{dy}{dx} = f(x, y)$.

$k_1 = h.f(x_k, y_k)$

$k_2 = h.f(x_k + h/2, y_k + k_1/2)$

$k_3 = h.f(x_k + h/2, y_k + k_2/2)$

$k_4 = h.f(x_k + h, y_k + k_3)$

$y_{k+1} = y_k + (1/6).(k_1 + 2k_2 + 2k_3 + k_4)$]

Question 9

A square plate of side 3 cm has a symmetrically placed square cavity of side 1 cm. The plate is charged to a potential 1 V on the inside and earthed outside and has no charges leaving or entering the plate. Obtain a system of linear equations to provide a numerical solution for the potential distribution on the plate, by taking a uniform grid of size ½ cm. How will you solve such a system in the easiest possible manner? Discuss how will the potential at the grid points change if the potential at the inside boundary is made 10 V.
