THE INSTITUTE OF ENGINEERS-SRI LANKA

PART III(A) EXAMINATION- OCTOBER 2013

301 MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) State Caucy-Reimann equations for an Analytic function f(z) = u + iv, where u and v are real variable functions of x and y.

(b) Prove that $u = x^2 - y^2 - 2xy - 2x - y - 1$ is harmonic function. Find the conjugate harmonic function v. Hence find an analytic function $f(z) = u + iv$. Express f(z) in terms of z, where $z = x + iy$.

(c) Define $f : C \to C \setminus \{2\}$ by $w = f(z) = \dfrac{2z-1}{z-2}$

Show that f maps the unit circle |Z| onto the unit circle |w|=1

Question 2

(a) Find all points at which $f(z) = \dfrac{x}{x^2+y^2} - i\dfrac{y}{x^2+y^2}$ is differentiable. At what points is f analytic?

Explain.

(b) Evaluate the following integrals using Cauchy Integral formula;

$\oint_C \dfrac{4-3z}{(z-1)^2(z-2)} dz$, where c is the circle $|z| = \dfrac{1}{2}$.

(c) Use Cauchy Residue theorem to find the value of the integral: $\int_0^{2\pi} \dfrac{d\theta}{3+2\cos\theta}$

Question 3

(a)Find the half range Fourier Sine series of the function $f(x) = \begin{cases} -1 & \text{for } 0, < x < \pi/2 \\ 1 & \text{for } \pi/2 < x < \pi \end{cases}$

(b) Find the function $u(x, t)$, defined for $0 \le x \le \pi$ and $t = 0$, which satisfies the initial-value problem:

$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$,

$u(0,t) = u(\pi,t) = 0$

$u(x, 0) = h(x)$,

where $h(x) = \begin{cases} x & \text{for } 0 \le x \le \pi/2 \\ \pi - x & \text{for } \pi/2 \le x \le \pi \end{cases}$.

Question 4

A researcher thinks there is a link between a person's height and level of confidence. She measured the height h, to the nearest cm, of a random sample of 9 people. She also devised a test to measure the level of confidence c of each person. The data are shown in the table below.

h	179	169	187	166	162	193	161	177	168
c	569	561	579	561	540	598	542	565	573

(a) Draw a scatter diagram to illustrate these data.

(b) Find exact values of S_{cc}, S_{hh}, S_{ch}

(c) Calculate the value of the correlation coefficient for these data.

(d) Give an interpretation of your correlation coefficient.

(e) Calculate the equation of the regression line of c on h in the form $c = a + bh$.

(f) Check whether b is significant to the model.

Question 5

(a) A random sample of 10 electrical components produced by a machine was tested for length of useful life and the data were recorded as follows;

Life in months 21.1 22.5 22.0 20.8 25.7 23.9 21.4 22.2 22.9 23.6

(i) Determine a 95% confidence interval for the mean life of this type of components.

(ii) Test the hypothesis that the mean life time of this type of component is at least 24 months at 5% level of significance.

(b) A construction company has three suppliers for four types of under-floor insulation material. The number of units of each type of insulation material supplied by the three suppliers is summarised in the following contingency table.

	A	B	C
Poly E	22	17	20
Poly S	35	29	42
M board	13	14	33
Seal F	10	20	45

A chi-squared test is carried out to test whether there is an association between suppliers and the type of insulation material supplied.

(i). State the null and alternative hypotheses.

(ii). The value of the test statistic calculated from the data was 16.97. Carry out the test yourself at 5% level of significance.

iii. State your conclusions carefully.

Question 6

A manufacturer wished to compare the wearing qualities of two different types of automobile tires, A and B, and he had 5 cars available for use in an experiment. To make the comparison, one tire of Type A and one of Type B were mounted on the rear wheels of each of the five automobiles.

(For each car, a coin was tossed to decide which tire would be mounted on the left side and which would be mounted on the right.). The automobiles were then operated for a specified number of miles and the amount of wear was recorded for each tire. These measurements appear below:

Automobile	Tire A	Tire B
1	10.6	10.2
2	9.8	9.4
3	12.3	11.8
4	9.7	9.1
5	8.8	8.3

Test an appropriate procedure is to be used for testing the null hypothesis that there is no difference in the average wear for the two types of tires at 5% level of significance.

Question 7

Given an initial value problem $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$, the second order Runge-Kutta method gives an approximate value for y at $x_1 = x_0 + h$ as $y(x_0) = y_0 + \frac{1}{2}(k_1 + k_2)$, where

$k_1 = hf(x_0, y_0),\ k_2 = hf(x_0 + h, y_0 + k_1)$.

Write down a similar scheme for solving a pair of non-linear differential equations.

Use this method to solve the system of equations: $\frac{dy}{dx} = z - xy,\ \frac{dz}{dx} = 2y - 2;$

$y(0) = 2$ and $z(0) = 1$ to find y and z at $x = 0.1$ and $x = 0.2$

For the same initial value problem determine $y(0.1)$ using the Taylor series method.

Question 8

The steady state temperature distribution over the region x >0 , y > 3, satisfies the partial differential equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

And the boundary conditions $\phi(0,y)=30,\ \ \phi(3,y)=0,\ \ \phi(x,0)=15,\ \ \phi(x,3)=30-10x,$

(i) Use standard approximation

$\frac{\partial^2\phi}{\partial x^2}=\frac{\phi(x+h,y)-2\phi(x,y)+\phi(x-h,y)}{h^2}$ and similar approximation for $\frac{\partial^2\phi}{\partial y^2}$ with step length 1 in each direction to formulate a set of equations for the approximate values of ϕ on the mesh in the Figure 1 below using the indicated numbering to denote the ϕ values.

(ii) Starting from initial values of ϕ, apply two iterations to the Gauss Seidel method to your equations.