The frequency-response method of analysis and design is a powerful technique for the comprehensive study of a system by conventional methods. Performance requirements can be readily expressed in terms of the frequency response.

Since noise, which is always present in any system, can result in poor overall performance, the frequency-response characteristics of a system permit evaluation of the effect of noise.

For a given control system, the frequency response of the controlled variable is

\[ G(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)} \]

Frequency response is a function of the pole-zero pattern in the s-plane.

**Frequency-Response Curves**

The frequency-domain plots belong to two categories.

1. **Bode plot** of the magnitude of the output-input ratio vs. frequency in rectangular coordinates.
2. **Nyquist plot** of the output-input ratio may be plotted in polar coordinates with frequency as a parameter.

**Log magnitude** The logarithm of the magnitude of a transfer function \( G(j\omega) \) expressed in decibels is

\[ 20 \log |G(j\omega)| \text{ dB} \]

This quantity is called the log magnitude, abbreviated \( \text{Lm} \).

**Decibels (dB)**: The unit commonly used for the logarithm of the magnitude.

**Octave and decade**

An octave is a frequency band from \( f_1 \) to \( f_2 \), where \( f_2/f_1 = 2 \).

There is an increase of 1 decade from \( f_1 \) to \( f_2 \) when \( f_2/f_1 = 10 \).

The typical transfer function

\[ G(j\omega) = \frac{K}{(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_3)\ldots} \]

Frequency response has two parts, magnitude and phase angle.

**Magnitude**

\[ \text{Lm}(G(j\omega)) = \text{Lm}(K) + \text{Lm}(1 + j\omega T_1) + \text{Lm}(1 + j\omega T_2) + \ldots \text{Lm}(1 + j\omega T_n) \]

A very attractive advantage of using logarithmic plot for the frequency response of transfer function is that multiplication of magnitudes contributed by individual factors can be converted into addition.
The amplitude plot may be approximated by two straight lines, (i) one with a slope of $\pm 20 \text{db/decade}$ and passing through frequency $\omega = 1/T$, known as corner frequency, (ii) the other is coincident with 0 db line. The actual plot approaches asymptotically to these straight lines and has a maximum error of $\pm 3 \text{db}$ occurring at the corner frequency. It is to be noted that the positive sign in the above and subsequent relations are considered for positive power of the factor whereas the negative sign are to be taken for negative power of the factor.

Also when $\omega \approx 1/T$, (at corner frequency), the phase angle is 45°. 

Bode plot for $F(s) = (1 + Ts)$ 

Bode plot of $F(s) = 1/(1 + Ts)$
Take transfer function as, $G(s) = \frac{50,000(s+10)}{(s+10)(s+500)}$

Convert the transfer function into time constant form, $s = j\omega$

$G(j\omega) = \frac{50,000 \cdot j\omega / (10+1)}{500 \cdot j\omega / (j\omega + 1) / (j\omega/500 + 1)} = \frac{100 \cdot j\omega / (10+1)}{(j\omega + 1)(j\omega/500 + 1)}$

Identify the individual components in the transfer function

Draw the bode plot for each component separately

Add them all to get the final form of bode plot

Find bode plot for $G(s) = \frac{1000}{(1 + 0.1)(1 + 0.001)}$

First draw the bode magnitude and phase plot for individual parts separately.

Bode plot for $G_1(s) = 1000$

Bode plot for $G_2(s) = \frac{1}{s}$

Bode plot for $G_3(s) = \frac{1}{(1 + 0.1)}$

Bode plot for $G_4(s) = \frac{1}{(1 + 0.001)}$

Find bode plot, when $G(s)H(s) = \frac{1000}{(s+0.5)(s+0.1)(s+0.01)}$

First draw the bode magnitude and phase plot for individual parts separately.