The frequency-response method of analysis and design is a powerful technique for the comprehensive study of a system by conventional methods. Performance requirements can be readily expressed in terms of the frequency response.

Since noise, which is always present in any system, can result in poor overall performance, **the frequency-response characteristics of a system permit evaluation of the effect of noise.**

For a given control system, the frequency response of the controlled variable is

$$C(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)} R(j\omega)$$

frequency response is a function of the pole–zero pattern in the *s-plane.*

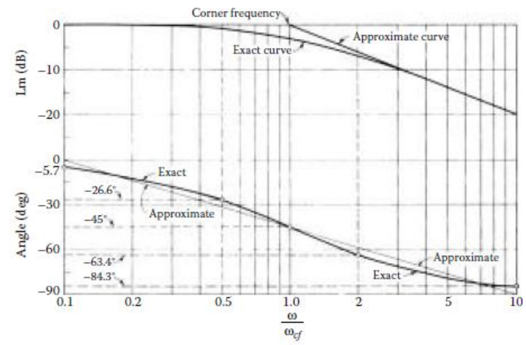
Frequency-Response Curves

The frequency domain plots belong to two categories.

(1) plot of the magnitude of the output–input ratio vs. frequency in rectangular coordinates. **(Bode plot)**

(2) output–input ratio may be plotted in polar coordinates with frequency as a parameter **(Nyquist plot)**

Typical bode plot for transfer function



The complex function G(*jω)H(jω) may be represented by two separate graphs, one* giving the magnitude versus frequency and the other the phase angle (in degrees) versus frequency.

Bode plot permits a simple method for obtaining an approximate log-magnitude curve of the transfer function. It is based on **straight-line asymptotic approximations**, which is adequate for the rough sketch of the frequency response characteristics needed in the design stage. And when exact curves are needed, corrections can be easily incorporated to these basic asymptotic plots.
The use of logarithmic scale makes it possible to display both the low- and high-frequency

it is convenient to use Bode diagram for plotting the frequency-response data obtained experimentally for a system for the estimation of transfer function.

Log magnitude *The logarithm of the magnitude of a transfer function G(jω) expressed in decibels is*

$$20 \log |G(j\omega)| \text{ dB}$$

This quantity is called the *log magnitude, abbreviated Lm.*

$$\mathrm{Lm}G(j\omega) = 20 \log |G(j\omega)| \text{ dB}$$

Decibels *(dB). The unit commonly used for the logarithm of the magnitude.*

Octave and decade

An octave is a frequency band from *f1 to f2, where f2/f1 = 2.*

There is an increase of 1 decade from *f1 to f2 when f2/f1 = 10.*

Some Common Numbers

Number	Decibels
0.01	−40
0.1	−20
0.5	−6
1.0	0
2.0	6
10.0	20
100.0	40
200.0	46

The typical transfer function

$$G(j\omega) = \frac{K_m(1 + j\omega T_1)(1 + j\omega T_2)^r \cdots}{(j\omega)^m(1 + j\omega T_a)\left[1 + (2\zeta/\omega_n)j\omega + (1/\omega^2)(j\omega)^2\right]\cdots}$$

Frequency response has two parts, **magnitude** and **phase angle**

Magnitude

$$\mathrm{Lm}G(j\omega) = \mathrm{Lm}K_m + \mathrm{Lm}(1 + j\omega T_1) + r\mathrm{Lm}(1 + j\omega T_2) + \cdots - m\mathrm{Lm}j\omega - \mathrm{Lm}(1 + j\omega T_a) - \mathrm{Lm}\left[1 + \frac{2\zeta}{\omega_n}j\omega + \frac{1}{\omega^2}(j\omega)^2\right] - \cdots$$

A very attractive advantage of using logarithmic plot for the frequency response of transfer function is that <u>**multiplication of magnitudes contributed by individual factors can be converted into addition**</u>

The phase angle,

$$\angle[G(j\omega)] = \angle[K_m] + \angle[1 + j\omega T_1] + r\angle[1 + j\omega T_2] + \cdots - m\angle[j\omega] - \angle[1 + j\omega T_a] - \angle\left[1 + \frac{2\zeta}{\omega_n}j\omega + \frac{1}{\omega^2}(j\omega)^2\right] - \cdots$$

The angle equation may be rewritten as

$$\angle[G(j\omega)] = \angle[K_m] + \tan^{-1}\omega T_1 + r\tan^{-1}\omega T_2 + \cdots - m90^\circ - \tan^{-1}\omega T_a - \tan^{-1}\frac{2\zeta\omega/\omega_n}{1 - \omega^2/\omega_n^2} -$$

In any transfer function, the numerator and denominator in following forms,

$$K_m$$
$$(j\omega)^{\pm m}$$
$$(1 + j\omega T)^{\pm r}$$
$$\left[1 + \frac{2\zeta}{\omega_n}j\omega + \frac{1}{\omega_n^2}(j\omega)^2\right]^{\pm p}$$

Once the logarithmic plots of these basic factors are known, it will be convenient to add their contributions graphically to get the composite plot of the multiplying factors of G(*jω)H(jω),*since the product of terms become additions of their logarithms.
It will be discovered soon that in the logarithmic scale, the actual amplitude and phase

Real constant gain K:

$$F(j\omega) = K$$

$$F_{db} = 20 \log_{10} K$$

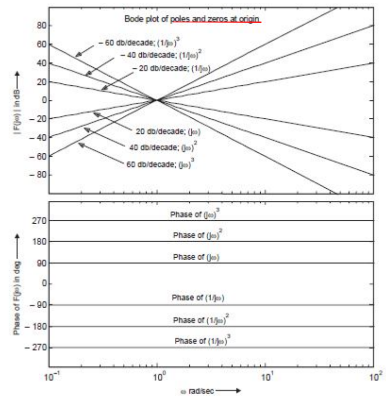
$$\angle F = \begin{cases} 0, & : K > 0 \\ -180^\circ & : K < 0 \end{cases}$$

Pure integral and derivative factors (Pole and Zero at the origin):

$$F(j\omega) = (j\omega)^{\pm n}$$

The magnitude $\quad F_{db} = \pm\, n\ 20 \log_{10}\omega \quad \text{for } \omega > 0$

$$\angle F = \pm\, n \times 90$$



(C) Factors corresponding to simple poles and zeros: $F(j\omega) = (1 + j\omega T)^{\pm 1}$

The log magnitude of the first-order factor is given by

Amplitude $\quad F_{db} = \pm\, 20 \log_{10} \sqrt{1 + (\omega T)^2}$

$$= \begin{cases} \pm 20 \log_{10} \omega T, & \omega T \gg 1 \\ 0 & , \quad \omega T \ll 1 \end{cases}$$

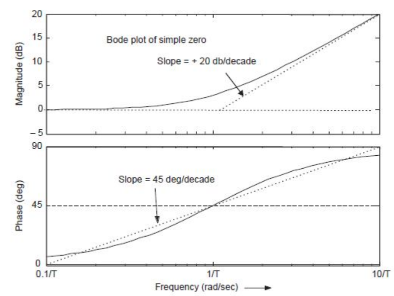
The amplitude plot may be approximated by two straight lines, (*i) one with a slope of* ± 20 db/decade and passing through frequency **ω = 1/T, known as corner frequency**,
(*ii) the other is coincident with 0 db line.*
The actual plot approaches asymptotically to these straight lines and has a maximum error of ± 3 db occurring at the corner frequency. It is to be noted that the positive sign in the above and subsequent relations are considered for positive power of the factor whereas the negative sign are to be taken for negative power of the factor.

The phase angle of first-order factor is given by:

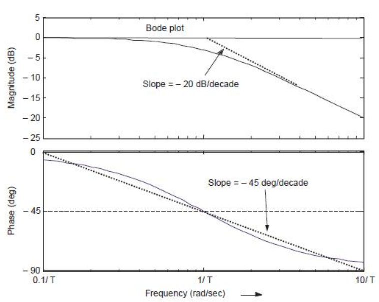
$$\angle F(j\omega) = \pm \tan^{-1}(\omega T)$$

$$= \begin{cases} 90^\circ & , \text{ for } \omega T \gg 10 \\ 0^\circ & , \text{ for } \omega T \ll 0.1 \end{cases}$$

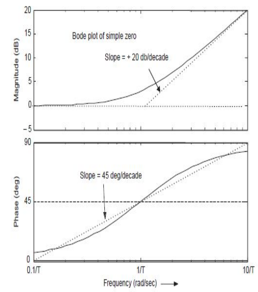
Also when w=1/T, (at corner frequency), the phase angle is 45^0



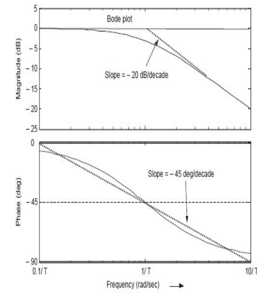
Bode plot for F(s) = (1 + Ts)



Bode plot of F(s) = 1/(1 + Ts)

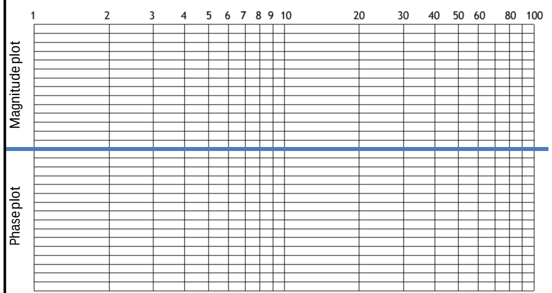


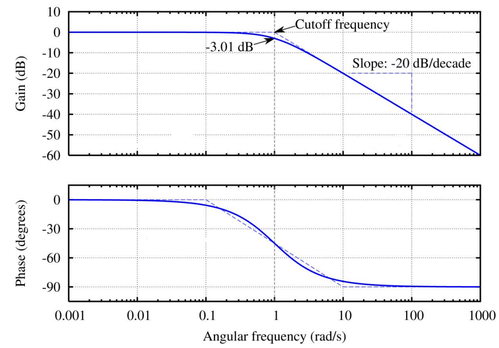
F(s) = (1+Ts)



F(s) = 1/ (1+Ts)

Semi – log graph paper





Take transfer function as, $G(s) = \frac{50,000(s+10)}{(s+1)(s+500)}$

Convert the transfer function into time constant form, s = jw

$$G(jw) = \frac{50,000x10(jw/10+1)}{500(jw+1)(jw/500+1)} = \frac{100(jw/10+1)}{(jw+1)(jw/500+1)}$$

Identify the individual components in the transfer function

Draw the bode plot for each components separately

Add them all to get the final form of bode plot

Find Bode plot for G(s) = $\frac{1000}{s(1+0.1s)(1+0.001s)}$

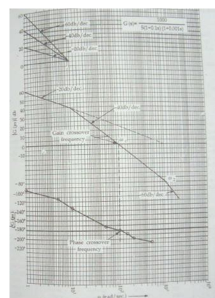
First draw the bode magnitude and phase plot for individual parts separately.

Bode plot for G1(s) = 1000

Bode plot for G2(s) = $\frac{1}{s}$

Bode plot for G3(s) = $\frac{1}{(1+0.1s)}$

Bode plot for G4(s) = $\frac{1}{(1+0.001s)}$



Find Bode plot, when G(s).H(s) = $\frac{16(1+0.5s)}{s^2(1+0.125s)(1+0.1s)}$