THE INSTITUTE OF ENGINEERS-SRI LANKA

PART III(A) EXAMINATION- OCTOBER 2008

301 MATHEMATICS

Answer FIVE Questions only **Time Allowed: Three Hours**

Question 1

(a) Prove that $u = x^2 - y^2 - 2xy - 2x - y - 1$ is a harmonic function. Find the conjugate harmonic function v. Hence find an analytic function $f(z) = u + iv$.

(b) Express f(z) in terms of z, where $z = x + iy$.

(c) Find the image and draw a rough sketch of the mapping of the region $1 \le x \le 2$ and $2 \le y \le 3$ under the mapping $w = e^z$.

Question 2

(a)Evaluate the following integral using Cauchy Integral formula;

$\oint_C \frac{4-3z}{(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{1}{2}$.

(b) Hence or otherwise, find $\oint_C \frac{4-3z}{(z-1)(z-2)^2} dz$, where c is the circle $|z| = \frac{1}{2}$.

(c) Use Cauchy Residue theorem to find the value of the integral: $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta} d\theta$

Question 3

(a) A tire manufacturer has been making tires for over 40 years. His best tire has averaged 52,500 miles with a standard deviation of 7,075 miles. A new tread design is thought to add additional wear to the tires. Sixty tires with the new design are tested, revealing a mean of 54,112 miles, with a standard deviation of 7,912 miles. Can it be said that the new tread adds to tire wear at 5% level of significance?

(i) State Hypotheses H_0 and H_1.

(ii) Calculate critical values.

(iii) Determine the decision rule.

(iv) State the conclusion of the text regarding rejecting or accepting of null hypothesis.

(b) The results of a study by the American Marketing Association to determine the relationship between the importance store owners attach to advertising and the size of store they own are shown in Table 1. Determine if there is any association between size of store and the advertising at 5% level of significance. Clearly state the hypotheses H_0 and H_1.

TABLE 1

	Advertising		
Size	Important	Not Important	No Opinion
Small	20	52	32
Medium	53	47	28
Large	67	32	25

Question 4

TABLE 2

Land Values	7	6.9	5.5	3.7	5.9	3.8	8.9	9.6	9.9	10
Cost of the House	67	63	60	54	58	36	76	87	89	92

The residents of a small town are worried about a rise in housing costs in the area. The mayor thinks that home prices fluctuate with land values. Data on 10 recently sold homes and the cost of the land on which they were built are shown in Table 2 in thousands of dollars. Identify the dependent and the independent variable. Find correlation between land value and cost of building a house. Construct and interpret the regression model. $y = \alpha + \beta x + e$.

On this basis, does it appear that the mayor is correct?

Question 5

(a) The price of a certain programmable calculator at several different stores is \$279.95, \$299.95, \$270.00, \$309.00, \$249.00, \$249.95, and \$229.98.

(i) Find the mean and standard deviation for these prices.

(ii) Suppose that a manufacturer's discount coupon for \$15 can be applied toward the price of the calculator. What is the mean and standard deviation for the discounted price?

(b) Car Ownership A survey of households selected randomly from a population recorded the number of cars per household. The results of the survey are indicated in the probability distribution as shown in Table 3.

TABLE 3

Number of cars, x	0	1	2	3	4	5
P(x)	0.10	0.28	0.39	0.17	0.04	0.02

Question 6

(a) Prove that if $X(f)$ is the Fourier Transform of $x(t)$ then $\int_{-\infty}^{\infty}(x(t))^2\,dt = \int_{-\infty}^{\infty}(X(f))^2\,df$

(b) Find the Fourier Transform of $x(t) = \begin{cases} 1 \ ; & -a < t < a \\ 0 \ ; & otherwise \end{cases}$

(c) Use the results in (a) and (b) to evaluate the integral $\int_0^{\infty}\frac{\sin^2 x}{x^2}dx$

Question 7

(a) Classify the Partial Differential Equation $\frac{\partial^2 u}{\partial x^2} = 4\,\frac{\partial u}{\partial t}$

(b) Briefly explain as to when we can use Fourier Series, Laplace Transform and Fourier Transform methods to solve a Partial Differential Equation.

(c) Find the solution of $\frac{\partial^2 u}{\partial x^2} = 4\,\frac{\partial u}{\partial t}$ with boundary conditions

$u(0,t) = u(1,t) = 0,$

$u(x,0) = \sin(\pi x)$

by a suitable method.

Question 8

Given an initial value problem $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$, the second order Runge-Kutta method gives an approximate value for y at $x_1 = x_0 + h$ as $y(x_1) = y_0 + \frac{1}{2}(k_1 + k_2)$, where $k_1 = hf(x_0, y_0),\ k_2 = hf(x_0 + h, y_0 + k_1)$.

Write down a similar scheme for solving a pair of non-linear differential equations.

Use this method to solve the system of equations: $\frac{dy}{dx} = z - xy,\ \frac{dz}{dx} = 2y - 2;$

$y(0) = 2$ and $z(0) = 1$ to find y and z at x = 0.1 and x = 0.2

For the same initial value problem determine y(0.1) using the Taylor series method.

Question 9

Explain under what conditions could explicit or implicit schemes be applied to find numerical solution to a parabolic partial differential equation with given initial and boundary conditions.

A metal bar scaled to unit length is heated to an initial temperature distribution given by $u(x,0) = 0.4x$, where x is the distance from one end. If the two ends are kept at temperatures, u =1 at x = 0 and u = 0 at x =1 ; for all t > 0, find numerically the temperature u(x, t) for $x = \frac{i}{4},\ i = 1, 2, 3$ and $t = \frac{j}{64},\ j = 1, 2.$